Precalculus Learning Goals - Week 4

This week we'll continue some more of our section on **Functions** – **Properties** and **Examples**. We'll finally finish it in Week 5.

As a reminder, the general goals for the section Functions - Properties and Examples are as follows. At the end of this section, students should be able to:

- Be comfortable with the language, notation, and pictures of functions, as well as be able to translate between them.
- Know several examples of functions and their basic properties, both mathematical and "real-world."
- Be able to generate new functions from old through the standard function operations.

More specifically, at the end of this week you should be able to:

- Compute an arithmetic combination of or composition of two or more functions based on a formula, graph, table, written description, or other presentation.
- Write a function as the composition of two simpler functions.
- Construct a rule to define the domain of a composition of functions based on the domains of the individual functions.
- Define 1-1 or *injective* functions and *inverse* functions.
- Explain why a function must be 1-1 in order to have an inverse.

Sample Problems. Here are some sample problems, of the type that you would do to demonstrate that you've learned the material. These are not the only types of problems you may see – they're just a sample.

- Let $f(x) = \frac{3}{x+2}$ and $g(x) = \frac{1}{2-x}$. Find a formula for f + g and $f \cdot g$ and simplify if possible. Compute $(f \circ g)(8)$. Find a formula for $f \circ g$ and $g \circ f$.
- Write the function $h(x) = \frac{x^2+3}{(x^2-1)^3}$ as a composition of two functions, f and g, such that $h = f \circ g$.
- Let g be given by the following graph (graph omitted), and let h be given by the following table (table omitted). Compute $(g^{-1} \circ h)(2)$.
- If the domain of f is $(3, \infty)$ and the domain of g is $(-\infty, 2) \cup (2, \infty)$, can you tell me what the domain of f + g is? What about f/g? What about $f \circ g$? If not, why not?
- Explain why a function must be 1-1 in order to have an inverse.
- T or F: $g(x) = \sqrt{x}$ is the inverse of $f(x) = x^2$.
- Let $f(x) = \frac{42x-57}{173x+12}$. What is $(f \circ f^{-1})(783)$?
- If a function passes the horizontal line test, does its inverse also pass the horizontal line test?